Procedures for Early-Stage Naval Ship Design Evaluation of Dynamic Stability: Influence of the Wave Crest[1]

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Abstract

A method is presented to assess stability changes in waves in early-stage ship design. The method is practical: the calculations can be completed quickly and can be applied as soon as lines are available. The intended use of the described method is for preliminary analysis. If stability changes that result in large roll motion are indicated early in the design process, this permits planning and budgeting for direct assessments using numerical simulations and/or model experiments. The main use of the proposed method is for the justification for hull form shape modification or for necessary additional analysis to better quantify potentially increased stability risk. The method is based on the evaluation of changing stability in irregular seas and can be applied to any type of ship. To demonstrate the robustness of the method, results for ten naval ship types are presented and discussed. The proposed method is shown to identify ships with known risk for large stability changes in waves.

1. Introduction

Since the end of the Cold War, the U.S. Navy has consistently sought to utilize more capability from fewer ships. Current and future needs will continue to require operations in a wide range of geographic regions and expand mission capabilities into more severe sea conditions. Because of these capability requirements, new ship platforms will continue to be considered, some with a significant departure from historical hull form geometries.

Despite numerous technological advancements, some areas of ship performance assessment still do not adequately evaluate the ship in the intended operational environment. Additionally, caution must be exercised in the design process to avoid dominated solutions, resulting in undesirable limitations in ship performance.

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[2] This paper expresses the personal views of the authors, which are not necessarily the official views of the Department of the Navy.

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14. ABSTRACT

A method is presented to assess stability changes in waves in early-stage ship design. The method is practical: the calculations can be completed quickly and can be applied as soon as lines are available. The intended use of the described method is for preliminary analysis. If stability changes that result in large roll motion are indicated early in the design process, this permits planning and budgeting for direct assessments using numerical simulations and/or model experiments. The main use of the proposed method is for the justification for hull form shape modification or for necessary additional analysis to better quantify potentially increased stability risk. The method is based on the evaluation of changing stability in irregular seas and can be applied to any type of ship. To demonstrate the robustness of the method, results for ten naval ship types are presented and discussed. The proposed method is shown to identify ships with known risk for large stability changes in waves.

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1.1 Stability Assessment

The assessment of ship stability remains an essential component of determining safety and operational effectiveness for all ship types. Static stability criteria (Sarchin & Goldberg, 1962) have been used for many decades for new ship designs. However, these criteria are empirically based on a population of ships which no longer reflect the multitude of geometries that exist in the current and expected future fleet. Once a design has departed far enough from that population, the inherent reliability provided through similarity with the populations of ships used to develop existing stability criteria may no longer be sufficient to assess dynamic stability.

Ship dynamics phenomena, not adequately covered by static stability criteria, present a challenge for both ship designers and regulators. For these cases, direct methods of analysis of the design, including extensive numerical simulations and model experiments are the only option available for performance assessment. Because some recent hull form designs have resulted in a radical departure from the population of ships in the database used for the development of current stability criteria, direct methods are being developed (Alman, et al., 1999; Ayyub, et al., 2006; Belenky, et al., 2008).

However, these methods are not simple and require significant investment in terms of time and cost. Therefore, a need exists to use simplified, yet practical, assessment tools early in the design process, to evaluate if dynamic stability failure is likely, and justify the application of direct methods.

This paper considers early-stage design assessment of one of the modes of dynamic stability failure; i.e., the decrease of stability caused by prolonged exposure to a wave crest.

1.2 Stability Changes in Waves

The stability of a ship is a measure of her ability to respond to an external heeling moment. This response is the resultant of some component of hydrostatic and hydrodynamic pressures applied to the submerged portion of the hull. The stability depends on the shape of this submerged portion of the hull. When a ship is subjected to the action of waves the submerged hull geometry is different from the static waterline. This is due to the distortion of water surface and the motions of the ship.

These changes are most pronounced when a ship is located on the crest or a trough of a wave, which is comparable to ship length (Figure 1). For most ships, the waterline becomes narrower forward toward the bow at the design waterline and wider at deeper waterlines when a ship is designed with bow flare.

When a wave trough is amidships, relatively wide sections fore, for a ship with bow flare, and aft are submerged and the actual waterline becomes wider in comparison to calm water. The stability is improved in this condition. However, when a wave crest is near amidships, the waterline intersects the slender parts of the hull lines forward and aft. The narrower waterline leads to a decrease in stability. Patterns of waterplane changes in waves may be different for a more unconventional hull shape.

When a ship is sailing in following or stern quartering seas, the encounter frequency decreases, and she may spend a significant amount of time riding near the wave crest. Prolonged time in this condition can lead to a loss of ship stability (Figure 2). The *GZ* curve on a wave crest may be insufficient to resist the heeling moment, while the

GZ in calm water is sufficient. As a result, the GZ curve becomes a function of both heel angle and time (see Figure 2 insert).

The phenomenon of decreased stability in waves, commonly referred as "pure loss of stability," was known to naval architects since the late 1800s (Pollard & Dudebout, 1892; Krylov 1958). It was uncommon to calculate the change of stability in waves until the 1960s. The first calculations were completed by Paulling (1961) and evaluated with a series of model tests by Necheav (1978; available in English from Belenky & Sevastianov 2007). As a distinct mode of stability failure, this phenomenon was identified during the model experiments in San-Francisco Bay (Paulling, et al., 1972, 1975; summary available from Kobylinski & Kastner 2003).

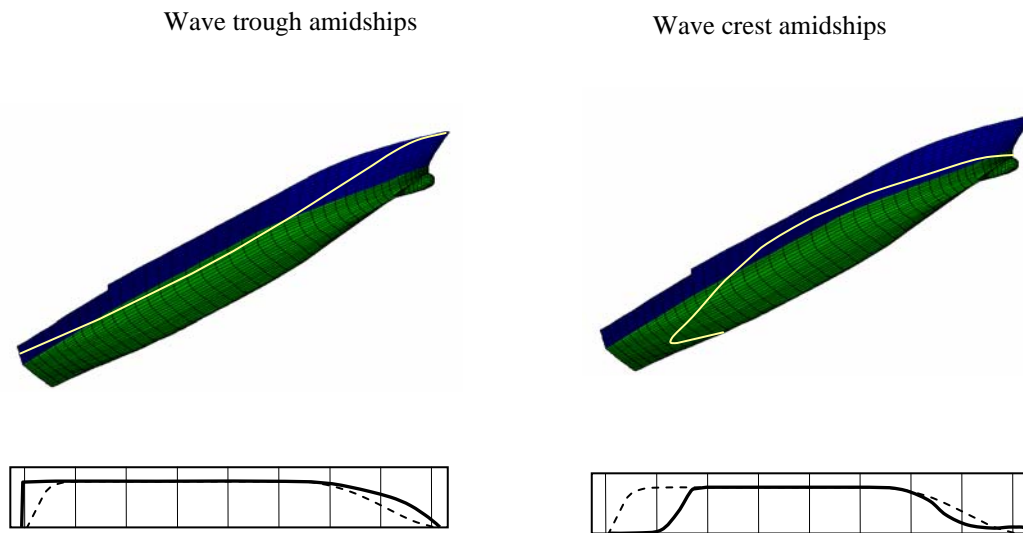Wave trough amidships   Wave crest amidships

Figure 1. Change of waterplane area for a naval ship (ONRFL) with conventional flared configuration and shallow draft stern
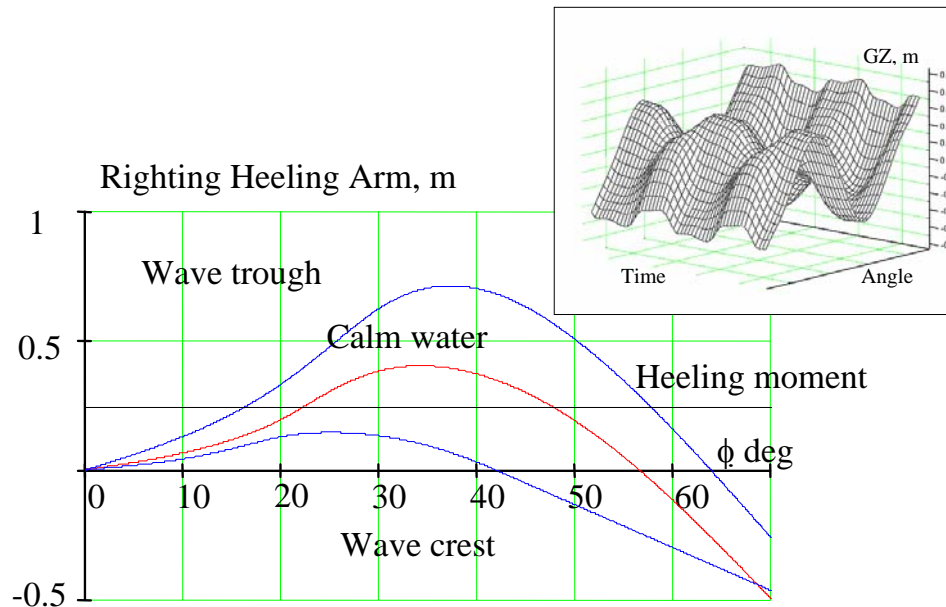
Figure 2. Change in the righting arm (GZ) curve due to the position of a ship on a wave

2. Method for Evaluation in Early-Stage Design

No method currently exists to evaluate stability changes in waves early in the design process. The method described in this paper is an attempt to address this critical need and a robust, yet relatively simple, analysis method for this evaluation is proposed.

For practical assessment of a ship in realistic operating conditions, stability change is affected by irregularity of waves and is therefore, a random event. As with any other stability failure, it involves large amplitude motions and, as a result, any practical assessment will require numerical simulation, and/or series of model experiments. These methods give the best answer that today's technology can provide, but they are quite expensive, both in terms of funding and time.

Time-domain numerical simulations were used for stability study at the end of the twentieth century (de Kat & Paulling, 1989), (de Kat, et al., 1994). Further development has led to a series of stability-related applications for naval combatants; see (Alman, et al., 1999). Upon the maturing of advanced hydrodynamic codes (Beck & Reed 2001; Shin, et al., 2003), different procedures were developed to use these codes (Shin, et al., 2004; Ayyub, et al., 2006; Belenky, et al., 2008). Such developments were also motivated by dynamic stability accidents with commercial vessels, such as a parametric roll accident (France, et al., 2003), which led to the implementation of a formalized assessment procedure (ABS, 2004). This procedure consisted of preliminary assessment using susceptibility and severity criteria. If a vessel was found to be susceptible to a particular type of dynamic stability failure, model tests (roll decay) and numerical simulations were recommended. Results of the analysis were presented in the form of ship-specific on-board operator guidance.

Similar ideas are being pursued by IMO for the development of new generation intact stability criteria for commercial vessels. The framework of these future regulations (Annex 1, SLF 51/WP.2) covers failures related with change of stability in waves (pure-loss of stability and parametric roll), broaching, and dead ship conditions. More information on this framework is also available from Belenky, et al. (2008). A multi-tiered approach is envisioned, where preliminary analysis using vulnerability criteria is expected to be used. If the likelihood of one, or more, modes of failures related to dynamic stability is indicted by the vulnerability criteria, then direct assessment methods are applied. Assessment methods include the use of advanced hydrodynamic codes and/or model experiments. Current stability regulations are applied if dynamic stability failure was found to be unlikely.

A state-of-the-art review of possible methods for application to vulnerability criteria is available from Bassler, et al. (2009), while some ideas to overcome potential limitations from previous method were examined in Belenky, et al. (2009). These vulnerability criteria for dead ship conditions, broaching, parametric roll and pure-loss of stability are currently under development by the Intact Correspondence Group on Intact Stability, established by the Subcommittee on Stability and Load Lines and on Fishing Vessels Safety (SLF) of IMO. The current status of this development is reflected in the report of this corresponding group (SLF 52/3/1; SLF 52/Inf.2). Annex 6 of the latter document contains a description of a vulnerability criterion for pure-loss of stability, submitted by the Government of the Unites States.

The procedure described below represents an adaptation of this method for naval combatants. The procedure is intended to provide an indication of the severity, and

subsequent risk, of a dynamic stability failure. Presently, only transverse stability (pure-loss and parametric roll) problems are consider. However, future work must be undertaken to develop similar early-stage design assessment methods for lateral plane stability failures, such as broaching.

3. Early-Stage Design Assessment of Stability Changes in Waves

Because of instantaneous changes in waterplane area, this stability failure mode may be considered as a single wave event. Typically, the worst-case wavelength is close to the length of the ship, $\lambda/L \approx 1.0$. However, in order to account for the effect of ship size relative to the wave conditions, stability changes should be evaluated in irregular waves.

3.1 Model of Irregular Waves

The envelope method (Figure 3) is used to present the stochastic process of wave elevations, $\zeta(x,t)$:

$$\zeta(t,x) = A(t,x)\cos(\Psi(x) - \Phi(t)) \tag{1}$$

where $A(t,x)$ is a stochastic process of the envelope, which can also be considered as wave amplitudes; $\Psi(x)$ is a stochastic process of wave phases in space; and $\Phi(t)$ is a stochastic process of wave phases in time. This is similar to the approach presented by Longuet-Higgins (1957).

Changes of stability are instantaneous, therefore, consideration of time is not required and the wave elevation can be considered only as a spatial stochastic process:

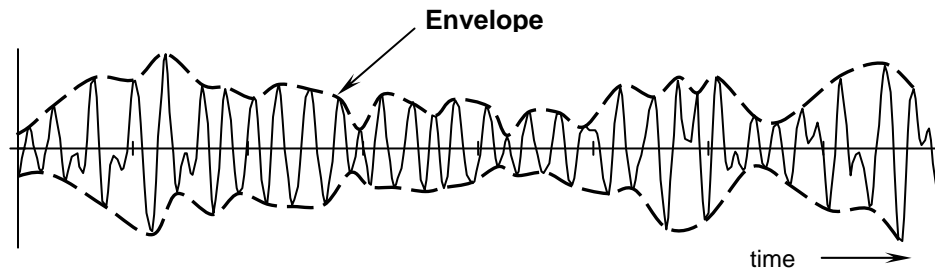$$\zeta(x) = A(x)\cos(\Psi(x)) \tag{2}$$

Figure 3. Envelope presentation of wave elevations

As a spectrum of wind-driven waves usually has a peak, which contains a significant part of all of the wave energy, the phase may be presented as

$$\Psi(x) = \Psi'x + \psi*(x) = k(x)x + \psi(x) \tag{3}$$

Here $k(x)$ is a slowly changing wave number associated with the absolute value of the spatial derivative of the phase Ψ'. The rest of the phase is presented as a stochastic process, $\psi*(x)$, or $\psi(x)$. These figures are not essential here: their role is to model the spatial autocorrelation function, where here only a single wave event is addressed.

To characterize an event of pure-loss of stability, the distribution of random wave numbers and wave amplitudes, $f(A,k)$, is needed.

$f(A,k) =$

$$\frac{A^2}{\sigma_\zeta^3 \sqrt{k_2^2 - k_1^2}\,\sqrt{2\pi}} \exp\!\left(-\frac{A^2}{2\sigma_\zeta^2}\right)\!\left(\exp\!\left(-\frac{A^2}{2\sigma_\zeta^2}\frac{(k-k_1)^2}{(k_2^2-k_1^2)}\right) + \exp\!\left(-\frac{A^2}{2\sigma_\zeta^2}\frac{(k+k_1)^2}{(k_2^2-k_1^2)}\right)\right) \qquad (4)$$

Where k_1 is the mean wave number, k_2 can be interpreted as a mean spectrum bandwith in terms of wave number, and σ_ζ is the standard deviation of wave elevations:

$$k_1 = \frac{1}{\sigma_\zeta^2} \int_0^\infty \frac{\omega^2}{g} s(\omega)d\omega \;\; ; \quad k_2 = \frac{1}{\sigma_\zeta} \sqrt{\int_0^\infty \frac{\omega^4}{g^2} s(\omega)d\omega} \qquad (5)$$

where g is the acceleration constant due to gravity. A more detailed explanation of the joint distribution is given in Appendix 1.

As a result of (4), the irregular seaway can be presented as a set of harmonic waves of random length and amplitudes. The probability of encounter of a wave with a certain height (amplitude) and length (wave number) can be calculated from the distribution (4).

Because stability change in waves is limited to waves comparable with ship length, it makes sense to limit consideration of the wave characteristics to the wave number corresponding to wave lengths of $0.5L$ and larger. The statistical weight of a wave with an amplitude A_i and wave number k_j is calculated as:

$$W_{ij} = \int_{A_i-\Delta A}^{A_i+\Delta A} \int_{k_j-\Delta k}^{k_j+\Delta k} f(A,k)dkdA \qquad (6)$$

Once the characteristics and statistical weight for a single wave have been defined, further stability calculations may be carried out as for regular waves.

3.2 Evaluation of Stability in Waves

The intended use of the method described previously is in early-stage design, where many different hull form variants may be analyzed. Therefore, the method must be sufficiently simple and require only a limited amount of information in order to assess the ship. It is also important to note that the objective for this early-stage design method is only to indicate increased susceptibility of stability failure, and provide a relative comparison, rather than numerically characterize it, which may be done using more

complex and detailed methods. Therefore, the stability evaluation is limited for this method by calculation of *GM* in waves.

A brief description of the procedure for the calculation of *GM* in waves is given in Appendix 2. In principle, it is not much different from calculation of *GM* in calm water. However, the most important difference is the necessity to find the equilibrium attitude of a ship. This enables the evaluation of the coordinates of the instantaneous waterline.

The *GM* value is calculated for each sinusoidal wave, with characteristics as defined in the previous section. These calculations are repeated for different positions of the wave crest along the ship length, so a complete wave pass is presented.

3.3 Formulation of the Assessment Procedure

For a stability assessment procedure to be effective, it must model the essential factors responsible for a stability failure.

The adverse influence of waves on stability is realized through a reduction of instantaneous waterplane, which results in a decrease of all stability characteristics. A ship with a conventional hull shape loses some stability when a wave crest is near amidships. However, some additional features of modern hulls are known to amplify this effect. One example is a buttock flow stern; shallow wide sections with relatively low transverse deadrise. Stern overhang is a typical feature of this type of stern. Because of the shallow draft of this type of stern, the entrance or exit from the water results in significant change of the waterplane for this section of the hull. Stability changes caused by stern overhang are also known to have the most influence on parametric roll of containerships (Levadou and van't Veer, 2006).

The configuration of the bow region also is important: bow flare increases area of the waterplane when a wave trough is located amidships and the ship is pitched, but is also capable of providing additional buoyancy for large angles of heel, limiting the likelihood of further heeling. A wave piercing bow has negative flare. The waterplane area decreases when the wave trough is amidships and the ship is pitched by the bow. Unlike ships with bow flare, no additional buoyancy is provided for large heel angles.

While the consideration of influence of hull geometry on pure-loss of stability is beyond the scope of this paper, it is clear that two main factors must be considered for any assessment: the change of waterplane, due to the passage of a wave, and the additional buoyancy, due to bow flare, that may provide additional compensation for heeling at large angles.

The *GM* change caused by the wave pass is an adequate description of stability in waves, because the changes of the waterplane are fully accounted for. Another factor that should to be included is the duration of time while stability is decreased. If this time is small the adverse influence of waves is not significant and there will be no time for a large roll angle to develop. This is shown in the insert in Figure 2, where changes of the *GZ* curve in time are shown in the form of 3-D surface. Calculation of the time while stability is decreased can be easily performed when the *GM* is considered as a function of the wave crest (formula A24). The critical level of *GM* needs to be established (discussed further below), then calculation of time while stability is decreased is trivial (Figure 4).
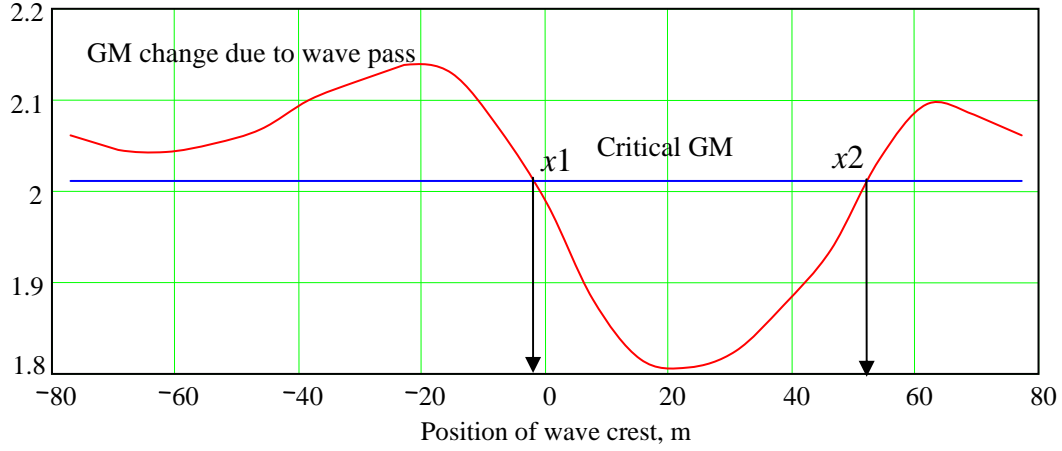
Figure 4. On the calculation of "time-below-critical-GM"

Points $x1$ and $x2$ in Figure 4 show the distance when the GM remains below the critical level, while the wave passes the ship. The "time duration below critical GM", tbc, can be calculated as:

$$tbc = \frac{x2 - x1}{c - V_S} \tag{7}$$

where c is wave celerity and Vs is ship speed. For irregular waves, both crossing points $x1$ and $x2$, as well as the wave celerity, are random numbers.

Using the model of irregular waves described above, the formula (7) can be considered as a deterministic function of random variables. In general terms, it can be expressed as:

$$tbc = \varphi(A, k) \tag{8}$$

A mean value of the deterministic function of two random variables is formally expressed as:

$$m(tbc) = \int\limits_{0}^{\infty} \int\limits_{-\infty}^{\infty} \varphi(A, k) f(A, k) dk dA \tag{9}$$

Note that because the wave number has been defined using the derivative of the spatial phase, it can take negative values. To calculate the mean value of time-below-critical-GM, statistical weights (6) are used:

$$m(tbc) = \sum_{i} \sum_{j} tbc_{ij} W_{ij} \tag{10}$$

As mentioned previously, the issue remains of how to assign a critical level of *GM*. As the *GM* due to the wave-pass takes care of waterplane changes, the critical *GM* has to take into account the features of hull form that can provide additional buoyancy at

large heel angles. The influence of these features, such a flared bow, is reflected in the position of the maximum of the calm-water *GZ* curve. This can be illustrated by a comparison of the *GZ* curve of two notional ships from ONR topside series (Bishop, et al., 2005). These ships have exactly the same hull shape below the calm-water waterline, but differ in topside configuration, one with flare and one with tumblehome. To illustrate the effect of topside configuration, Figure 5 shows the GZ curve calculated for the same value of *KG* for each of the two topside configurations.
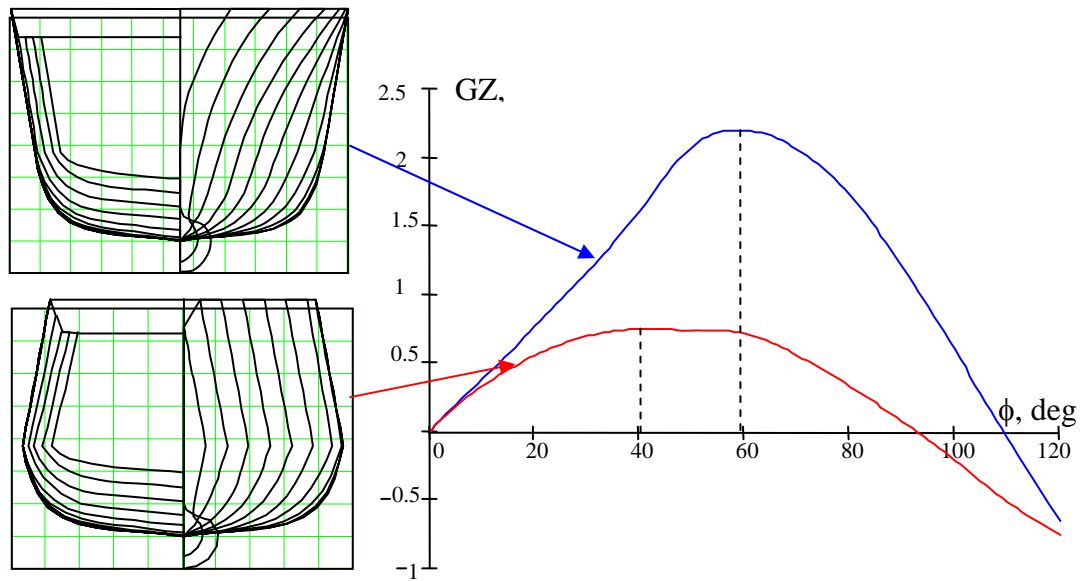

Figure 5. Geometries and the *GZ* curves of the ONR tumblehome and flared topside configurations (KG=7.5 m in both cases)

As shown in Figure 5, the angle of maximum *GZ* of the flared topside configuration is much larger than that of the tumblehome topside configuration. The difference in the value of the maximum of the *GZ* curve is even more dramatic. However, the angle of the maximum of the *GZ* curve is a preferable measure. Stability failure near a wave crest is a phenomenon occurring in a very small encounter frequency, while the wave crest is slowly moving along the hull. As a result, heeling may occur almost statically; so in this case, the angle of maximum represents the actual stability range. Therefore, setting the level of critical *GM*, based on the angle of the maximum, seems to be reasonable, because it takes into account the influence of large volumes of buoyancy that may be used as a stability reserve.

Using the angle of maximum as a basis to set the critical *GM* level also requires assigning the minimum positive *GM*, as the *KG* value corresponding to the critical angle of the maximum of the *GZ* curve may drive the *GM* into a negative area, leading to appearance of lolling that is considered unacceptable in normal operational conditions.

The assessment metric, C, is based on the ratio of the mean time below the critical GM, corresponding to the maximum of the righting arm, and the roll period corresponding to the critical GM at the maximum of the righting arm. This metric then

assesses the significance of stability change, because even though the instantaneous GM may decrease below the critical level, if it is for a short duration, then the effect of this event on the ship stability may not be significant. However, for longer duration decrease of stability below the critical level, the restoring moment may be degraded enough to result in a dangerously large roll angle. Because the assessment procedure must be usable for ships of any size and hull configuration, the final form of the assessment metric has to be non-dimensional:

$$C = \frac{m(tbc)}{T_\phi} ; \quad T_\phi = \frac{0.8B}{\sqrt{GM_{cr}}} \tag{11}$$

Where T_ϕ is natural period of small-amplitude roll in calm water, B is the maximum ship breadth, and GM_{cr} is the critical value of GM, determined as described above.

3.4 Sample Ships for Evaluation

Ten representative naval ship types were evaluated for stability performance in waves using the assessment procedure described above. These included a range of vessel sizes and classes, with representative hull form types from both the modern and near-term future fleet. These representative ship types include:

- Aircraft carrier
- Amphibious transport dock ship
- Guided missile cruiser
- Guided missile destroyer
- Frigate
- Coastal mine hunter
- Coastal patrol craft
- Semi-planing large corvette (SPLC)
- Modern destroyer-type hull form with tumblehome topside (ONRTH)
- Modern destroyer-type hull form with flared topside (ONRFL)

These ship types are representative of the composition of the modern USN surface fleet, with a few additional ship types, which are similar to those being considered for addition to the fleet in the near-term. Because these ships often have to travel as part of a task force, the stability in waves was evaluated for a speed of 15 knots, chosen to represent a nominal transit speed in heavy-weather conditions.

The ships also included the ONR tumblehome topside hull, which is known to be vulnerable to decreased stability near a wave crest (Bishop, et al., 2005; Bassler, et al., 2007; Hashimoto, 2009). This was intended to provide a verification of the assessment method.

3.5 Results of Calculations for the Sample Ships

The calculated assessment metric for each of the sample ships are shown in Table 1 and Figure 6. The values of KG for all the ships were set for critical angle of the maximum of 30 degrees, specified as a critical roll angle threshold, provided that GM not be less than 0.2 m. This assumed limitation of GM=0.2 m was introduced to model the existing requirements for a minimum metacentric height. The establishment of generalized value, if possible, will be addressed in future work. The ships were all evaluated for a speed of 15 knots. Calculations were performed for Sea State 4 through 8, defined with significant wave height H_s and modal period T_m. A Bretshneider spectrum was used. Discretization of the joint distribution (4) was performed with six values for amplitudes and thirteen values for wave numbers. The wave pass was modeled with twenty-one positions of the wave crest equally distributed along the ship length.

Table 1. Results of calculations for the sample ships

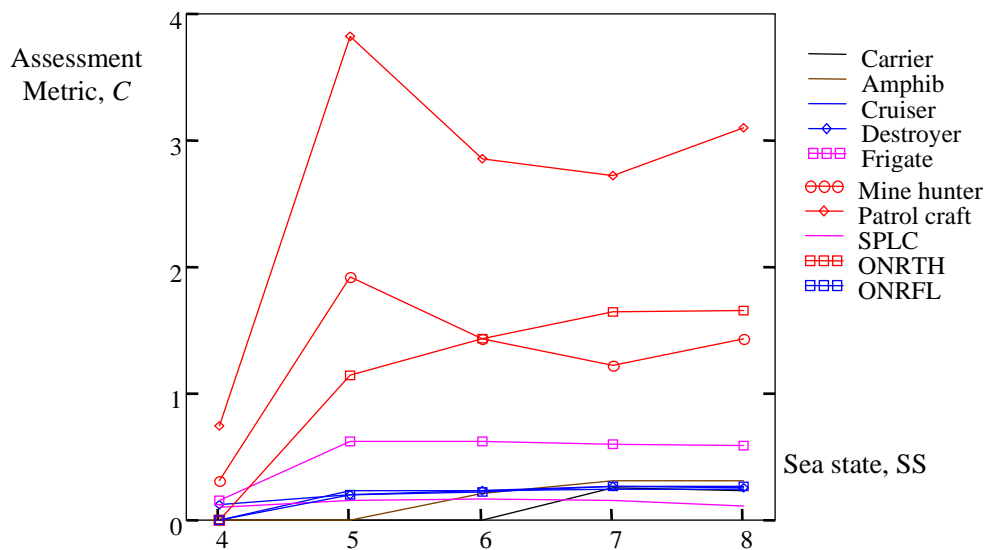| Sample Ship	GM_{cr}, m	SS 4 H_s=1.25 m T_m=8.8 s	SS 5 H_s=3.25 m T_m=9.7 s	SS 6 H_s=5.0 m T_m=12.4 s	SS 7 H_s=7.5 m T_m=15.0 s	SS 8 H_s=11.5 m T_m=16.4 s
Carrier	3.32	0	0	0	0.25	0.23
Amphib	0.82	0	0	0.20	0.30	0.31
Cruiser	0.20	0	0.23	0.23	0.23	0.24
Destroyer	0.20	0.12	0.20	0.23	0.27	0.25
Frigate	0.57	0.16	0.62	0.62	0.60	0.59
Mine hunter	1.15	0.30	1.92	1.42	1.22	1.42
Patrol craft	0.82	0.74	3.82	2.85	2.71	3.10
SPLC	0.20	0.09	0.15	0.16	0.15	0.10
ONRTH	1.66	0	1.14	1.43	1.64	1.65
ONRFL	0.20	0	0.19	0.22	0.26	0.26

Figure 6. Results of calculations for sample ships

3.6 Analysis of the Results

Among the sample ships evaluated, three of them stand out showing larger values for the assessment metric: the patrol craft, the mine hunter and the ONR tumblehome topside configuration. Both the patrol craft and the mine hunter are coastal ships that are not intended for blue-water transit in heavy-weather. As a result, they do not have much stability reserve and the critical *GM* is relatively large, while the corresponding period is relatively small. Both ships are also small in comparison with the waves for higher sea states, which mean that the relatively short waves are capable of degrading their stability beyond the critical level for a longer duration. Shorter waves have lower celerity, resulting in longer time when a ship is exposed to that wave. Therefore, the method points out that a smaller ship needs to have better stability than a larger one, in order to achieve similar stability in heavy sea conditions. This conclusion is well known from practical experience, and this result from the presented method illustrates that the method is correct.

For the ONR tumblehome topside configuration, the stability failure caused by pure-loss of stability is known from a number of numerical and experimental studies (Bishop, et al., 2005; Bassler, et al., 2007; Hashimoto, 2009). The fact that the present method was capable of determining the increased risk of pure-loss of stability for the ONRTH, compared to other ships of similar size, also demonstrates the robustness of the method.

The semi-planing large corvette had the lowest assessment metric for the higher sea state. This can be explained because the ship was designed to operate in littoral areas, but also be able to cross oceans with a carrier or expeditionary strike group. As a result, this ship has significant stability reserves with the angle of maximum of the *GZ* curve located at more than 60 degrees. This large reserve of stability drives down the value from the metric formula (11).

The other six ships have shown results rather close to each other. They represent a conventional set-up of a carrier or expeditionary strike group. Therefore, their dynamic stability characteristics are expected to be similar. The largest value among these six ship types belongs to the frigate. This is expected, due to its relatively small size. The likelihood of stability failure caused by wave crest for the cruiser, destroyer, and ONR flared hull configuration are almost identical, which is expected due to the similarities in hull shape and stability characteristics. The carrier and the amphib are the two largest ships that were examined. Their critical *GM* values are relatively large, but the waves capable of causing changes in their stability are rare. As a result the amphib experiences only small stability changes, beginning in Sea State 6, and the carrier beginning in Sea State 7.

In general, the results of calculations confirm existing experience for the ships considered, but also show differences in the dynamic stability in waves as a function of ship type and size.

4. Summary and Conclusions

This paper presents a method for early-stage ship design evaluation of stability changes in waves. Because of the departure from past, more traditional, hull form design, additional parameters must be considered for the assessment of ship performance in

intended operational environments. However, this often requires extensive analysis and model tests to determine the effects of hull form geometry changes, both above and below the waterline. A simple, yet robust method, for evaluation of stability changes in waves, will help to focus the additional costly analysis which may be required for radical departures in hull form shape.

The method described uses irregular waves of a given sea state. An encounter with a wave of particular length and height is considered as a random event, and its probability is evaluated. Changes in *GM* are calculated while a ship experiences a wave pass event. The time during this wave pass event while the value of *GM* is below critical level is evaluated. This time is then averaged for the representative populations of waves for a given sea state. The ratio of the averaged time below critical *GM* and natural roll period is the assessment metric used in the method to determine the likelihood of experiencing stability failure caused by a wave crest. The evaluation metric represents the ratio of the effect of the reserve buoyancy, resulting from hull shape, and the duration of time the ship may be susceptible to a large roll angle event.

Results of sample calculations are given for ten representative naval ship types. These results are consistent with existing experience, which demonstrates the technical correctness and robustness of the method.

The method is intended to be used as a preliminary evaluation tool in the framework of a multi-tiered approach for the evaluation of dynamic stability risk. By providing an early assessment of hull forms in common operational conditions, designers may be able to eliminate undesirable hull form shapes, or understand known risks of inferior designs which may be selected due to other operational requirements. Although the method has been applied only to monohulls, in principle there does not appear to be any principle limitations for extending this type of assessment method to multi-hull ships as well.

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Appendix 1 Joint Distribution of Waves Amplitudes and Numbers

Because wave elevations are known to have a normal distribution, then the joint distribution of the wave amplitudes and the derivatives of the phase is available from envelope theory:

$$f(A, \Psi') = \frac{A^2}{\sigma_\zeta^3 \sqrt{k_2^2 - k_1^2} \sqrt{2\pi}} \exp\left(-A^2 \frac{k_2^2 - 2k_1 \Psi' + \Psi'^2}{2\sigma_\zeta^2 (k_2^2 - k_1^2)}\right) \tag{A1}$$

where k_1 is the mean wave number and k_2 can be interpreted as a mean spectrum band in terms of wave number, σ_ζ, is the standard deviation of wave elevations,

$$k_1 = \frac{1}{\sigma_\zeta^2} \int_0^\infty \frac{\omega^2}{g} s(\omega)d\omega \; ; \quad k_2 = \frac{1}{\sigma_\zeta} \sqrt{\int_0^\infty \frac{\omega^4}{g^2} s(\omega)d\omega} \tag{A2}$$

where *g* is the acceleration constant due to gravity.

To derive the joint distribution of wave number and wave amplitudes, the derivative of the phase in (4) needs to be substituted by its absolute value. The absolute value of the derivative of the phase, Ψ', can be considered as a deterministic function of a random variable. Then the distribution of the wave number can be found from the conditional distribution of the derivative of the phase:

$$f(\Psi' \mid A) = \frac{f(A, \Psi')}{f(A)} = \frac{A}{\sigma_\zeta \sqrt{k_2^2 - k_1^2} \sqrt{2\pi}} \exp\left(-\frac{A^2}{2\sigma_\zeta^2} \frac{(\Psi' - k_1)^2}{(k_2^2 - k_1^2)}\right) \tag{A3}$$

The marginal distribution of amplitudes, *f(A)*, is the well-known Rayleigh distribution:

$$f(A) = \frac{A}{\sigma_\zeta^2} \exp\left(-\frac{A^2}{2\sigma_\zeta^2}\right) \tag{A4}$$

The function of absolute value is:

$$k = u(\Psi') = \begin{cases} \Psi' & if \quad \Psi' > 0 \\ -\Psi' & otherwise \end{cases} \tag{A5}$$

This function is not monotonic, but it has two monotonic sub-domains. Therefore, its distribution contains two components:

$$f(k) = f(v_1(k)) \,|\, v_1'(k)\,| + f(v_2(k)) \,|\, v_2'(k)\,| \tag{A6}$$

Here $v(k)$ is a function inverse to (A7) and v' is its derivative. Because the function u is not monotonic, its inverse expression is not single-valued, so two values exist at the same time for all k, and its derivative is also dual-valued:

$$v(k) = \begin{cases} v_1 = \Psi' \\ v_2 = -\Psi' \end{cases} \qquad v'(k) = \begin{cases} v_1' = 1 \\ v_2' = -1 \end{cases} \tag{A7}$$

Therefore,

$$f(k) = f(k) \,|\,{-}1\,| + f(-k) \,|\,1\,| = f(k) + f(-k) \tag{A9}$$

The application of (A9) for (A6) yields the conditional distribution of with wave number:

$$f(k \,|\, A) = f(k \,|\, A) + f(-k \,|\, A) =$$

$$\frac{A}{\sigma_\zeta \sqrt{k_2^2 - k_1^2}\,\sqrt{2\pi}} \left(\exp\left(-\frac{A^2}{2\sigma_\zeta^2} \frac{(k-k_1)^2}{(k_2^2 - k_1^2)} \right) + \exp\left(-\frac{A^2}{2\sigma_\zeta^2} \frac{(k+k_1)^2}{(k_2^2 - k_1^2)} \right) \right) \tag{A10}$$

Finally the joint distribution of wave number and amplitudes is expressed as:

$$f(A,k) = f(A)f(k \,|\, A) =$$

$$\frac{A^2}{\sigma_\zeta^3 \sqrt{k_2^2 - k_1^2}\,\sqrt{2\pi}} \exp\left(-\frac{A^2}{2\sigma_\zeta^2} \right) \left(\exp\left(-\frac{A^2}{2\sigma_\zeta^2} \frac{(k-k_1)^2}{(k_2^2 - k_1^2)} \right) + \exp\left(-\frac{A^2}{2\sigma_\zeta^2} \frac{(k+k_1)^2}{(k_2^2 - k_1^2)} \right) \right) \tag{A11}$$

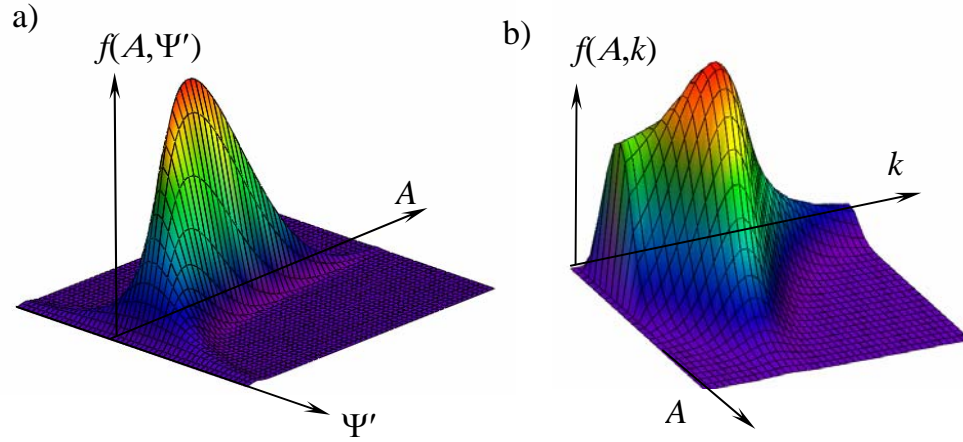Both distributions (A1) and (A11) are shown in Figure A1.

Figure A1. Joint distribution of amplitudes and derivatives of phases (a), amplitudes and wave numbers (b)

Appendix 2 Calculation of GM in Waves

The wave elevation along the ship length is defined as:

$$\zeta(x, x_C) = A\cos(k(x - x_c))$$ (A12)

The area at each station and its moment relative to the vertical axis are expressed as function of the local draft, accounting for the sinkage and the trim:

$$A_i(z) = 2\int_0^z b_i(z)dz \qquad Mz_i(z) = 2\int_0^z zb_i(z)dz$$ (A13)

where i indicates the station number, $b_i(z)$ is the half-breadth at station i, at the local draft, z. The volumetric displacement can be expressed as a function of the position of a wave crest for an array of local drafts $\vec{z} = \{z_i\}, i = 1, N_s$:

$$V(x_C, \vec{z}) = 0.5\sum_{i=1}^{N_{st}-1}\left(A_i(z_i) + A_{i+1}(z_{i+1})\right)(x_{i+1} - x_i)$$ (A14)

where x_i is the coordinate of the i-th station in the ship-fixed coordinate system.

The moments of the hull relative to vertical and longitudinal axes are expressed using a similar formulation:

$$MZ(x_C, \vec{z}) = 0.5\sum_{i=1}^{N_{st}-1}\left(Mz_i(z_i) + Mz_{i+1}(z_{i+1})\right)(x_{i+1} - x_i)$$ (A15)

$$MX(x_C, \vec{z}) = 0.5\sum_{i=1}^{N_{st}-1}\left(x_i A_i(z_i) + x_{i+1} A_{i+1}(z_{i+1})\right)(x_{i+1} - x_i)$$ (A16)

Formulae (A15) and (A16) can be used to express coordinates for the center of buoyancy:

$$LCB(x_C, \vec{z}) = \frac{MX(x_C, \vec{z})}{V(x_C, \vec{z})}; \qquad KB(x_C, \theta, z_S) = \frac{MX(x_C, \vec{z})}{V(x_C, \vec{z})} \qquad (A17)$$

The local draft at each station comes from formula (A12), describing wave elevations along the hull, and depends on sinkage and trim. To account for the trim on the wave profile, the following auxiliary function is introduced:

$$\Xi(x_i, z_i, \theta, x_C) = a_W \cos\left[k\left(x_i \cos\theta - z_i \sin\theta - x_C\right)\right] - \left(x_i \sin\theta + z_i \cos\theta\right) \qquad (A18)$$

This function equals zero when a point with coordinates x_i and z_i is exactly at the surface of the wave of amplitude A, rotated by the trim angle θ. Then the elevation of the wave profile at the i-th station is defined through the inverse of the function Ξ, calculated for each station located at x_i:

$$z_{WL}(\theta, z_S, x_i, x_C) = INV\left(\Xi(x_i, z_i, \theta, x_C)\right) + z_S \qquad (A19)$$

where z_S is the sinkage.

The wave profile along the ship hull is evaluated by satisfying equilibrium conditions, through solving the following system of nonlinear algebraic equations, with trim and sinkage as unknowns

$$\begin{cases} V(x_C, z_{WL}(\theta, z_S, x_i, x_C)) = V_0 \\ LCB(x_C, z_{WL}(\theta, z_S, x_i, x_C)) = LCB_0 \end{cases} \qquad (A20)$$

Once sinkage and trim are found, the profile of the wave along the hull can be found as:

$$z_i = z_{WL}(\theta, z_S, x_i, x_C) \qquad (A21)$$

The moment of inertia of the waterplane made by the wave profile is:

$$I_X(x_C) = \frac{1}{3} \sum_{i=1}^{N_{st}-1} \left(b^3(z_i) + b^3(z_{i+1})\right)(x_{i+1} - x_i) \qquad (A22)$$

Other hydrostatic terms are also needed to determine GM

$$KB(x_C) = \frac{MZ(x_C, \vec{z})}{V_0}; \quad BM(x_C) = \frac{I_X(x_C)}{V_0} \qquad (A23)$$

Finally the value of GM in waves is a function of the position of a wave crest

$$GM(x_C) = BM(x_C) - KG + KB(x_C). \qquad (A24)$$